جامعة تكريت

كلية الهندسة

قسمي الهندسة الكيمياوية والهندسة الكهربائية

الميكانيك الهندسي - ستاتك

الفصل الثاني

2019-2018

UNIVERSITY OF TIKRIT
ENGINEERING COLLEGE
Chemical & Electrical Engineering Department

Engineering Mechanics
Statics Lectures

Chapter two

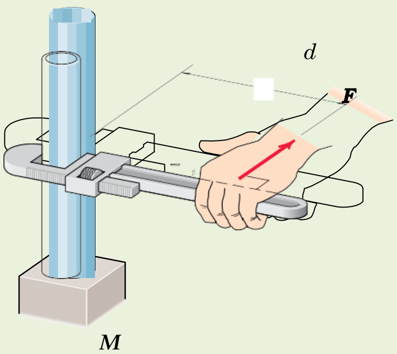
Lecturer
Sabah Mahdi salih

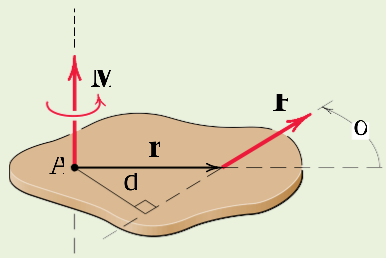
2018 -2019

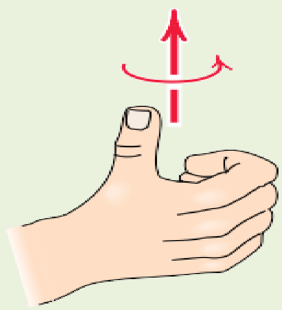
Chapter Two

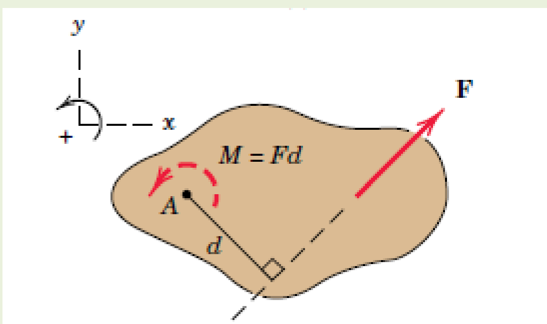
2 -1 The MOMENT :

In addition to the tendency to move a body in the direction of its application,a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.









(d)

Figures shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

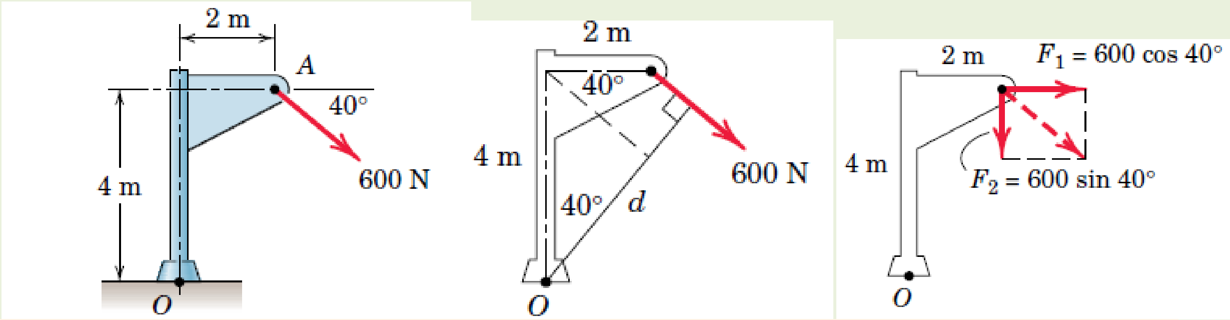
$$\boxed{M = Fd}$$

يمكن ايجاد قيمة العزم عن طريق عدة حلول وهنا في هذا المثال يمكن الحل باربع طرق يعتبر **اسهلها** الحل بنظرية فارينون والتي تنص على ان العزم الناتج عن قوة ما يساوي مجموع العزوم الناتجة عن مركبات هذه القوة والطريقة الثانية تسمى طريقة المسافة العمودية المباشرة (direct D)4 فتسمى نظرية نقل القوة على طول خط فعل تلك القوة. اما الطريقتين ال 3 و

2-1-1 Varignon's Theorem

One of the most useful principles of mechanics is Varignon's theo-rem, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

Example 1 : Calculate the magnitude of the moment about the base point O of the 600-N force in four different ways.



Solution. *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

① By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

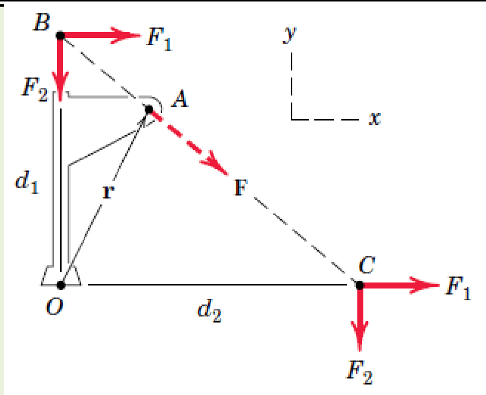
(II) Replace the force by its rectangular components at A

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

② $$M_O = 460(4) + 386(2) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes



$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

③ *(IV)* Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

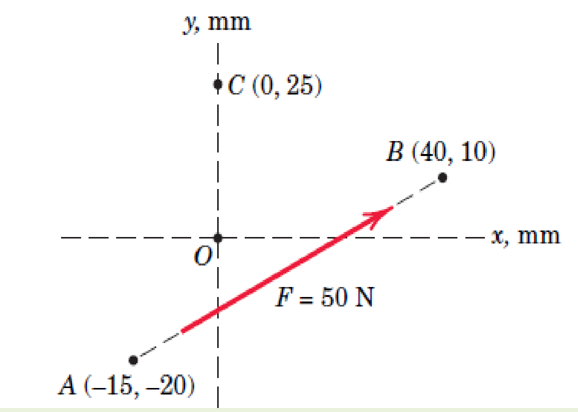
$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

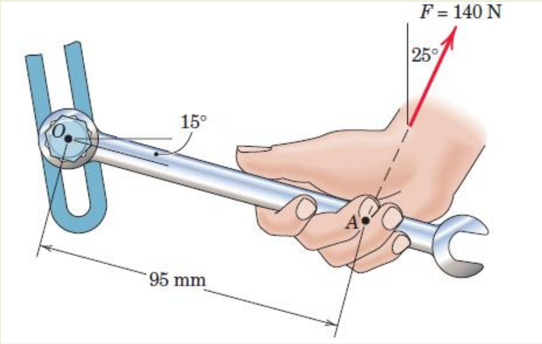
Example 2 : determine the Moment of 50 N force about (a) point *O* by varygnon theorem (b) point C by an other method.

Sol: ……



Example 3 : determine the Moment of force (140N) which exerted by wrench on a bolt about the bolt center (O).

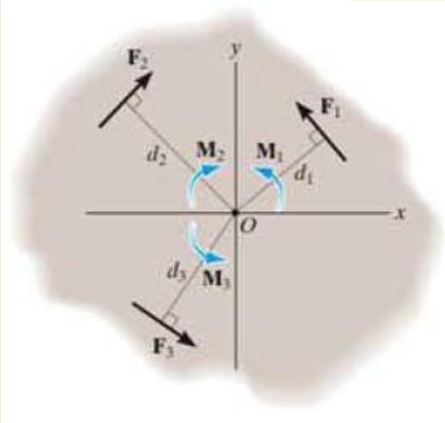
Sol: ….



2-2 Resultant moment

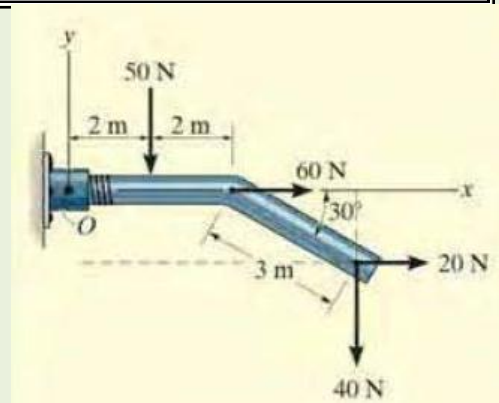
Resultant Moment. For two-dimensional problems, where all the forces lie within the x–y plane, Fig. 4–3, the resultant moment $(\mathbf{M}_R)_o$ about point O (the z axis) can be determined by *finding the algebraic sum* of the moments caused by all the forces in the system. As a convention, we will generally consider *positive moments* as *counterclockwise* since they are directed along the positive z axis (out of the page). *Clockwise moments* will be *negative*. Doing this, the directional sense of each moment can be represented by a *plus or minus* sign. Using this sign convention, the resultant moment in Fig. 4–3 is therefore

$$\zeta+ (M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$



Ex:3.

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O.



$$\circlearrowleft + M_{R_O} = \Sigma F d;$$

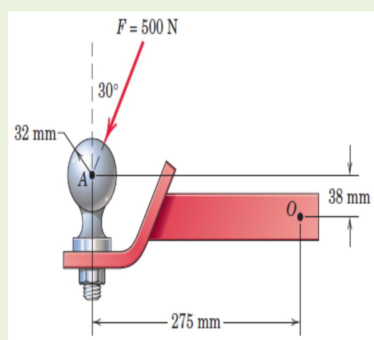
$$M_{R_O} = -50\ \mathrm{N}(2\ \mathrm{m}) + 60\ \mathrm{N}(0) + 20\ \mathrm{N}(3 \sin 30^\circ\ \mathrm{m})$$

$$-40\ \mathrm{N}(4\ \mathrm{m} + 3 \cos 30^\circ\ \mathrm{m})$$

$$M_{R_O} = -334\ \mathrm{N \cdot m} = 334\ \mathrm{N \cdot m} \circlearrowright$$

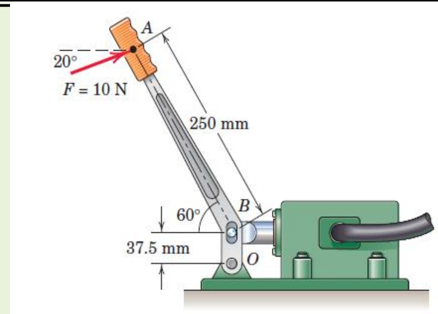
Problems: (1)

As a trailer is towed in the forward direction, the force $F = 500$ N is applied as shown to the ball of the trailer hitch. Determine the moment of this force about point O.

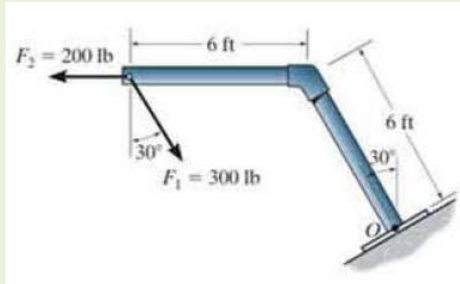


(2)

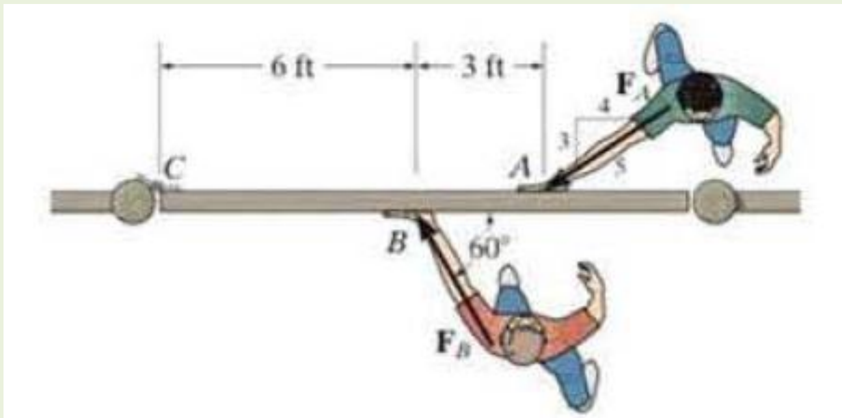
The 10-N force is applied to the handle of the hydraulic control valve as shown. Calculate the moment of this force about point O.



(3) find out the resultant moment of two forces shown in Fig. Down about point (O).



(4) Two boyes push on the gate in same time. If $F_{B} = 30$ N ,what is the value of the force F_A=?? the boy A must exert to prevent the gate to turning?



2-2 Moment of a couple .

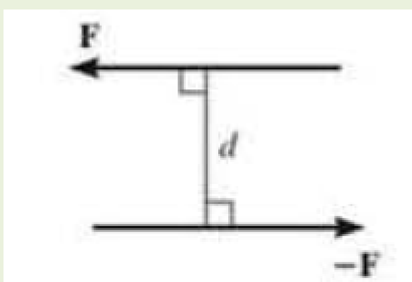
A *couple* is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, Fig. 4–25. Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate.

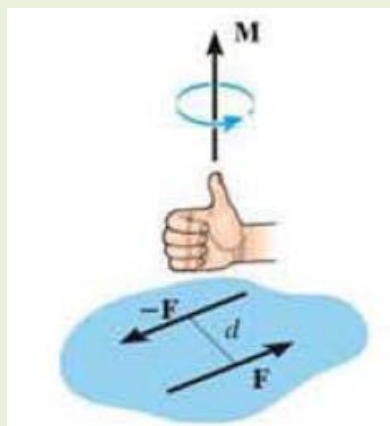
The moment produced by a couple is called a *couple moment*. We can determine its value by finding the sum of the moments of both couple forces about *any* arbitrary point. For example, in Fig. 4–26, position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ are directed from point O to points A and B lying on the line of action of $-\mathbf{F}$ and $\mathbf{F}$. The couple moment determined about O is therefore

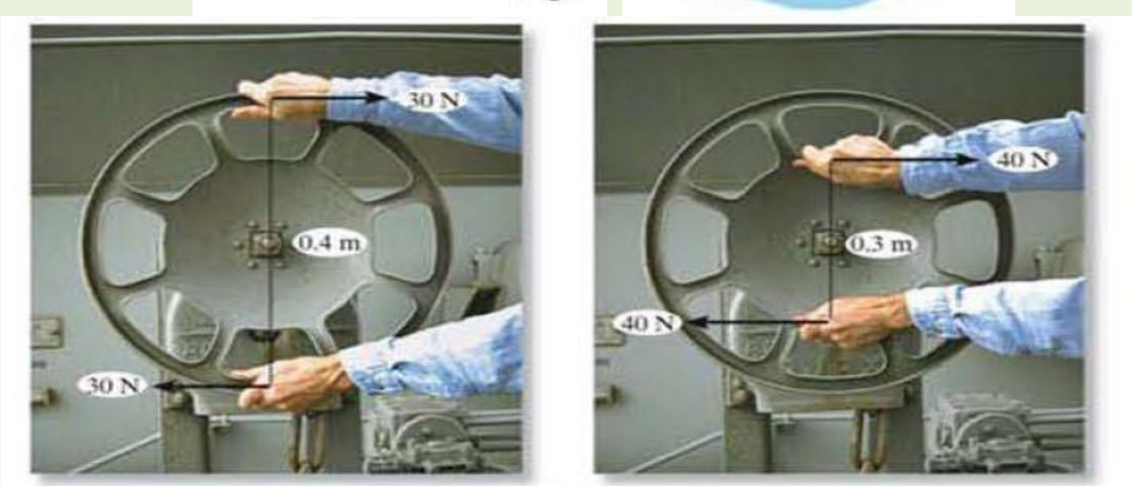
$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \tag{4–13}$$







Ex 2-2-1

EXAMPLE 4.10

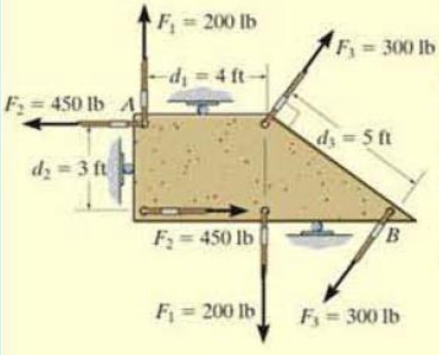


Fig. 4–30

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft + M_R = \Sigma M;\; M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

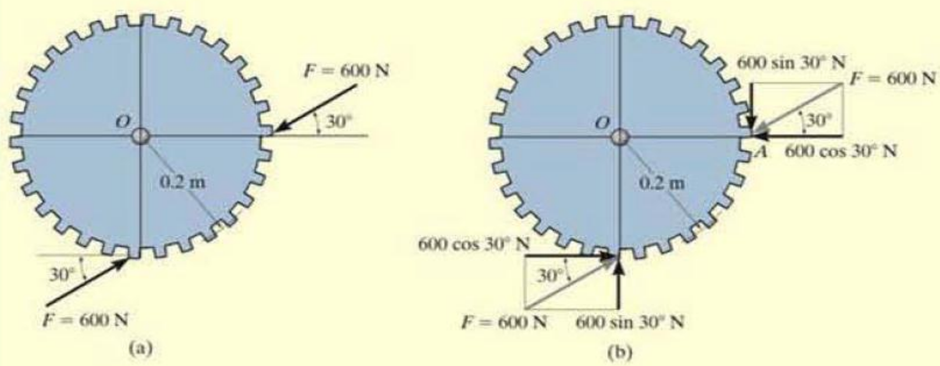
$$= (-200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

$$= -950 \text{ lb}\cdot\text{ft} = 950 \text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

Ex 2-2-2

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



Sol:….

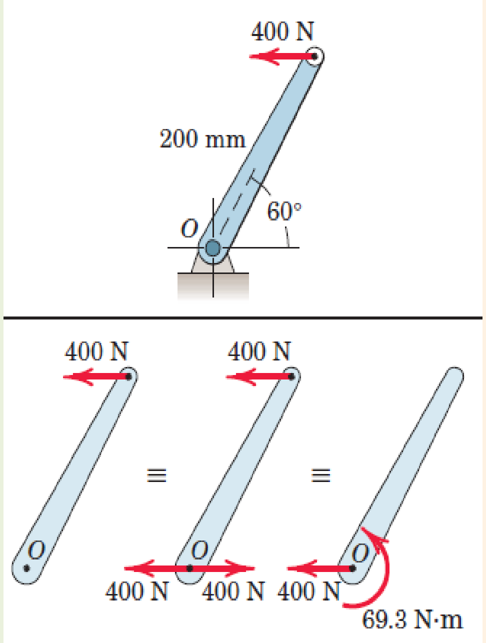
Ex 2-2-3

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.

Solution. We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

$[M = Fd]$ $\qquad M = 400(0.200 \sin 60^\circ) = 69.3\ \text{N·m}$ $\qquad$ *Ans.*

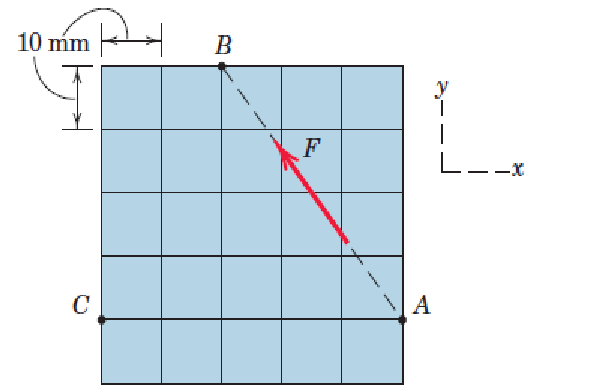
Thus, the original force is equivalent to the 400-N force at O and the 69.3-N·m couple as shown in the third of the three equivalent figures.



Problems:

(1)

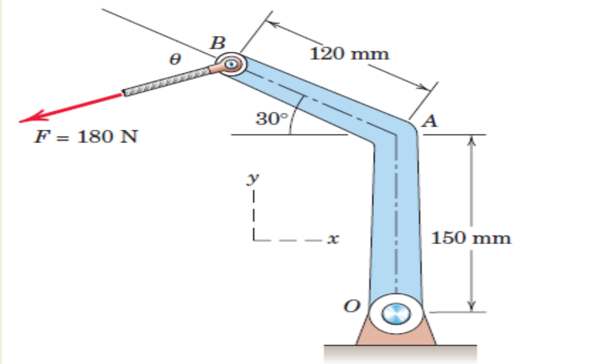
A force $F = 60$ N acts along the line AB. Determine the equivalent force–couple system at point C.



(2)

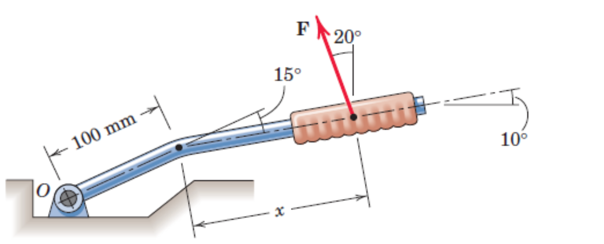
The 180-N force is applied to the end of body OAB. If $\theta = 50^\circ$, determine the equivalent force–couple system at the shaft axis O.

Ans. $\mathbf{F} = -169.1\mathbf{i} - 61.6\mathbf{j}$ N, $M_O = 41.9$ N·m CCW



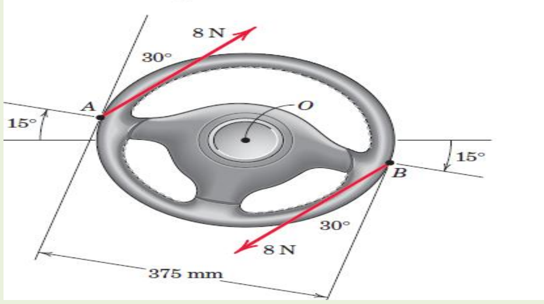
(3)

2/68 A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point O.



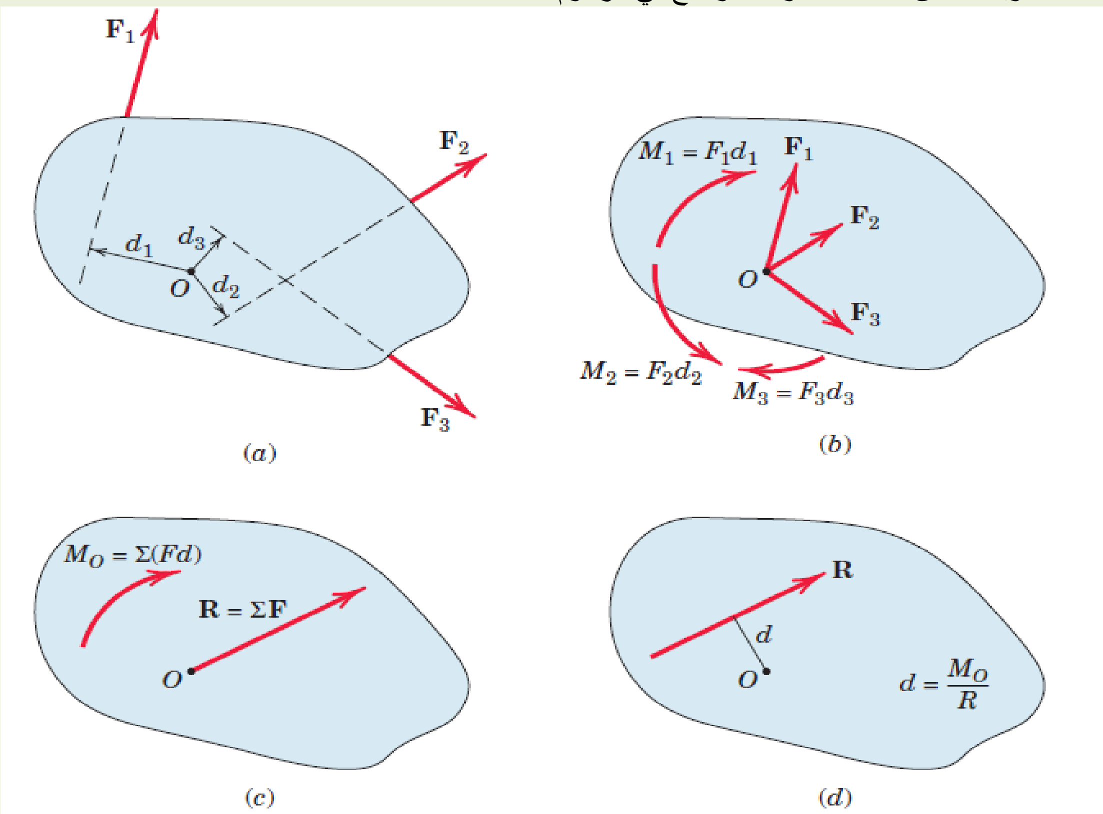
(4)

During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially-inward component. Determine the moment exerted about the steering column at O.



2-3 Resultant (forces &moments)

عندما تؤثر عدة قوى على جسم ما يمكن حساب العزم والمحصلة لتلك القوى حول نقطة ما وتلك النفقطة يتم تحديدها لتعتبر نقطة معروفة ليتم بعد ذلك تحديد موقع المحصلة وبعدها عن تلك النقطة وكما موضح في الرسوم ادناه.



حيث ان المحصلة يمكن حسابها وفق المعادلات الاتية:

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

وقيمة العزم وبعد المحصلة (موقعها) يمكن حسابه حسب الاتي:

$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

Ex 2-3-1: find the resultant of four forces and one couple shown in Fig. down and indicate the location with respect to (o) along X-axis.

Sample Problem 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.

Solution. Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x] \qquad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y] \qquad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}] \qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right] \qquad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*

① $[M_O = \Sigma(Fd)] \qquad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$= -237 \text{ N·m}$

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

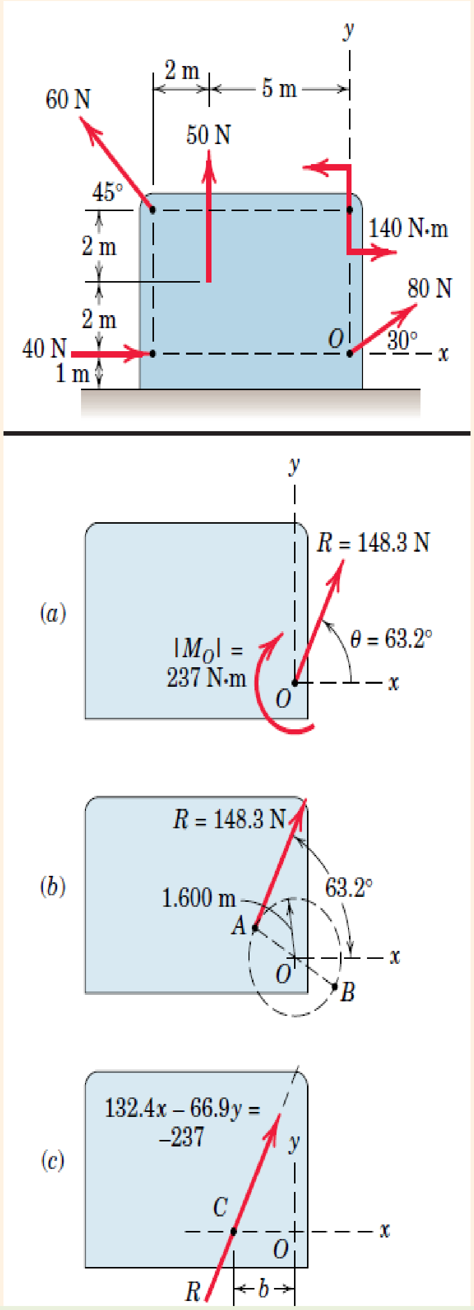
We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|] \qquad 148.3d = 237 \qquad d = 1.600 \text{ m}$ *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.600-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. *a*, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point B.

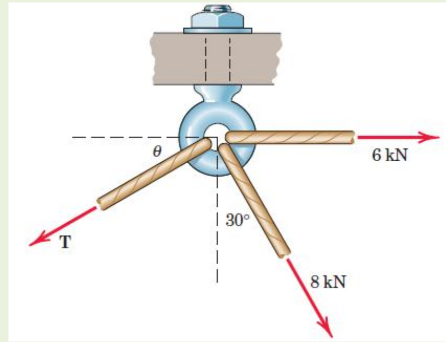
The resultant **R** may also be located by determining its intercept distance b to point C on the x-axis, Fig. *c*. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$



Ex 2-3-2: find the magnitude of T and its direction (ϴ) for which the eye bolt under a resultant downward of 15Kn

Sol…..

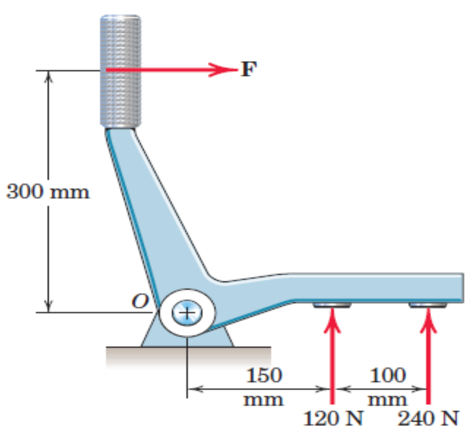


Notes….. ↑ → o ↗

Problems:

(1)

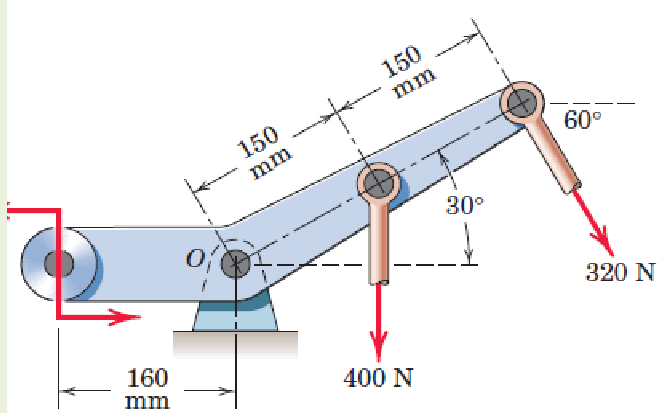
Determine the magnitude of the force **F** applied to the handle which will make the resultant of the three forces pass through *O*.



(2)

If the resultant of the two forces and couple *M* passes through point *O*, determine *M*.

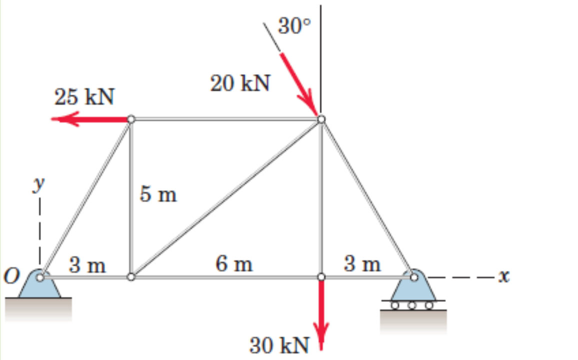
Ans. $M = 148.0$ N·m CCW



3

Determine the resultant **R** of the three forces acting on the simple truss. Specify the points on the x- and y-axes through which **R** must pass.

Ans. $\mathbf{R} = -15\mathbf{i} - 47.3\mathbf{j}$ kN
$x = 7.42$ m, $y = -23.4$ m



(4)

7 Replace the three forces acting on the bent pipe by a single equivalent force **R**. Specify the distance x from point O to the point on the x-axis through which the line of action of **R** passes.

Ans. $\mathbf{R} = -200\mathbf{i} + 80\mathbf{j}$ N, $x = 1.625$ m (off pipe)